

# Elliptical Mathematical Scholium of Petrophysical Modulation of Volume of Shale Paradigm from Linear and Non-Linear Confluxibilities

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**Abstract:** Shale characterization and volume prognosis using only the linear confluxibilities creates quantifiable petrophysical constraints on the volumetric illation of oil and gas reserves since it is an inverse homology to formation porosity. Although, linear correlations are quicker and most often used but are most often degraded by quantificational errors due to the degree of formation heterogeneity complexities, while its degree of linearity varies with severity of complex sandstone reservoirs such as shaly-sands and sandy-shales noticeable in the X-Field, Niger Delta, Nigeria. The non existence of non-linear correlations encapsulated in most logging software increases the degree of intricacies of the linear rubric and puts enormous strains on the magnitude of petrophysical based uncertainties. The aim of this study is to carry out a detailed comparative study of linear and non-linear homologies for astute prognosis of volume of shales from gamma ray well logs in a Field in the Niger Delta. The main objective is to develop new mathematical models/correlations for quicker quantitative reservoir elucidation and effective management of both linear and non-linear models of volume of shales from gamma ray logs. Quantitative and qualitative petrophysical interpretation techniques were comprehensively used for both the linear and 4 non linear correlations in 5 oil wells with 25 sand zones in the X-Field. Results of volume of shale showed collective convergent declining curves of all the non-linear models with increasing gamma ray content in contrast to declining straight line of linear model. Results of modified volume of shale for this field gave  $V_{shmod} = -0.46\ln(GR_{log}) + 1.997$  with square regression ( $R_2$ ) of 0.996. This perhaps shows increased volume of shale profile with depth. Model validation results showed good matches with  $R_2 > 0.9$  with Steiber 2 given as  $R_2 = 0.999$ , Steiber 3 given as  $R_2 = 0.991$ , average Steiber given as  $R_2 = 0.999$ , Larinov (older rocks) given as  $R_2 = 0.997$ , Larinov (tertiary rocks) given as  $R_2 = 0.999$ , Clavier given as  $R_2 = 0.999$  and IGR (gamma ray index) given as  $R_2 = 0.999$ . Detailed petrophysical re-evaluation of the non-linear models was achieved with perhaps appreciable square regression and a clear departure from the conventional linear model initially perceived as the most accurate. It is recommended that more studies be done with large data volume from regional Field studies to buttress the few points highlighted in this research.

**Keywords:** Shales, correlations, linear, non-linear, evaluation, petrophysics, sandstone reservoirs.

## Introduction

Shale is a clay-rich heterogeneous rock which contains variable content of clay minerals (mostly illite, kaolinite, chlorite and montmorillonite) and organic matter (Ejeh and Ideozu, 2018). The occurrence of shale in a pay zone has dire consequences on petrophysical evaluation and reduces effective and total porosity and permeability of the reservoir (Egu and Ilozobhie, 2021). This compounds quantificational analysis of uncertainties in formation evaluation and adequate estimation of oil and gas reserves (Egu, 2014). Shale distribution in a formation critically influences the evaluation of all principal reservoir characteristics e.g. effective porosity, water saturation, and permeability (Ali-Nandalal et al., 2010). Dispersed shale is composed of clay particles, fragments or crystals to be found on grain surface that occupy void spaces between matrix particles and reduce the effective porosity ( $\phi_e$ ) and permeability significantly (Christine et al., 2012). Structural shale exists in the form of fragments or crystals which are an integral part of the rock framework and is considered as a portion of rock matrix (David, 2021). Laminar shale exists as layer of shale which does not exceed 0.5 in. (1.27 cm) thickness within clean formations. The effect of two last shale types on porosity and permeability is assumed to be negligible (James and Oladiran, 2010; Egu 2020).

However, the terms such as 'volume of shale ( $V_{shale}$ )' and 'volume of clay ( $V_{clay}$ )', are applied in the calculations of water saturation in shale-bearing formations such as shaly sands. These are used interchangeably, assuming that they are the same, which they are not (Davud et al., 2018). Linear volume of shale estimation is a simple linear equation describing a straight line pattern for the determination of shale volume in pay zones (Eric et al., 2019). It is a function of the shale and sand base lines and gamma ray readings from the zones of interest (Les et al., 2014). Non linear correlation on the other hand is basically non linear equations used for the estimation of volume of shales due to the increasing shale content of the zone of interest (Egu and Ilozobhie, 2020). They are used to reduce the errors of the linear correlation (Mouin and Zulfiqar, 2019).

## Statement of problem

Shale characterization and volume estimation using only the linear correlation creates quantifiable petrophysical constraints on the volumetric estimates of oil and gas reserves since it is an inverse correlation to formation porosity (Ilozobhie and Egu, 2020). Linear correlations on the other hand are quicker and most often used but may be degraded by errors due its formation heterogenous complexities, while its degree of linearity varies with severity of complex sandstone reservoirs such as shaly-sands and sandy-shales. The non existence of non-linear correlations encapsulated in most logging software increases the degree of inadequacies of the linear approach and puts enormous strains on the magnitude of petrophysical based uncertainties. These daring technical challenges may perhaps reduce critical well log interpretation of lithologic qualities of complex reservoir particularly in the Niger Delta.

## Aim and Objectives

The aim of this erudition is to carry out a detailed study of linear and non-linear correlations for effective estimation of volume of shales from gamma ray well logs in a Field in the Niger Delta.

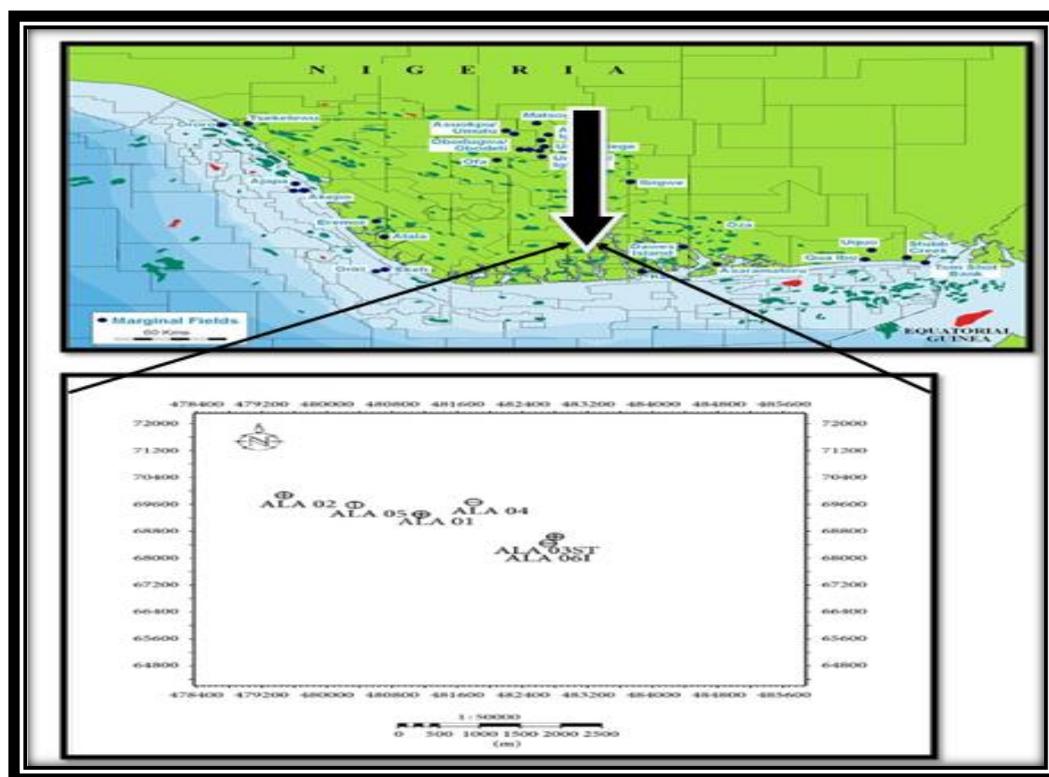
The objectives are;

- (i) To carry out a detailed qualitative interpretation of well logs from suites of logs provided for this Field.

- (ii) To carry out comparative quantitative analysis between linear and non-linear confluxibilities.
- (iii) To develop computer algorithm of the new mathematical models/correlations for quicker quantitative reservoir characterization and effective management of both linear and non-linear models of volume of shales from gamma ray logs applicable to the Field of study.
- (iv) To make recommendations where necessary for improved academic and technical enhancement of this study.

### Study area

The present study area is located between latitudes  $4^{\circ}.00^1$  and  $6^{\circ}.00^1$ N and longitudes  $5^{\circ}.00^1$  and  $7^{\circ}.00^1$  E at the onshore depobelt of Eastern Niger Delta, Nigeria (Fig. 1).



**Figure 1:** Map of the study area in the Niger Delta showing Oil Fields and Pipelines (After Ilozobhie and Egu, 2020)

### Materials and Method

#### Material

The data used is five composite well logs with 25 identified sand zones. It was supplied in digital form (compact disc) by Exxon Mobil Nigeria through the recommendation of the Department of Petroleum Resources (DPR). They include the following;

- **Composite well logs**

Composite well logs data provided for this work include; Gamma log, Resistivity log, Neutron log and Density log.

- **Gamma ray log**

Primarily, this log was used in lithology identification and boundary demarcation. It was provided for all five the wells.

- **Resistivity log**

This log was basically used for fluid identification. Resistivity log was provided for all the five wells

- **Neutron logs**

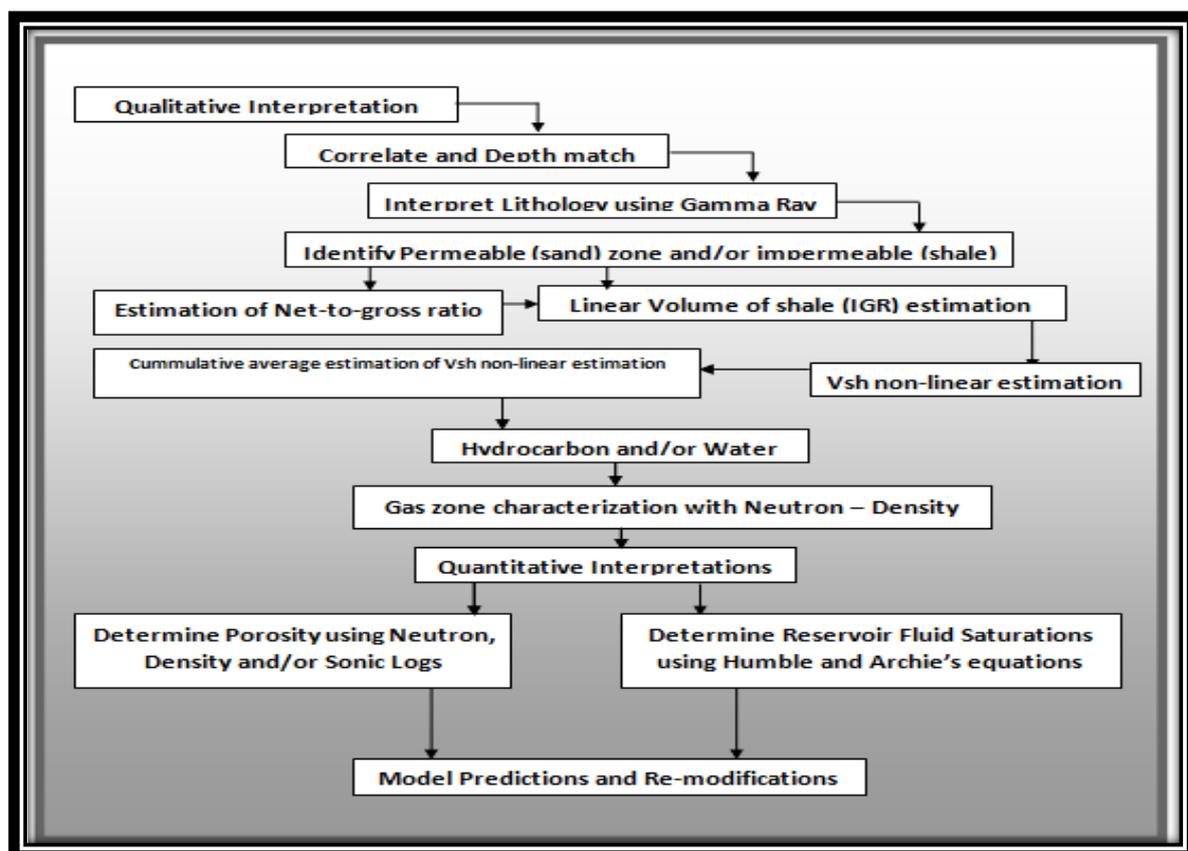
This log was used during the porosity logs crossovers (Neutron Density log crossovers) for fluid type determination. It was provided for all the five wells.

- **Density logs**

The Density log was used during the porosity logs crossovers (Neutron Density log crossovers) for fluid type determination. It was provided for all five wells.

## Methods

The data made available was in digital form and the interpretation and analysis of interpretation in this study was visually done. Fig. 2 shows the work flow of the different methods that were used in the course of this study.



**Figure 2:** A detailed flowchart of method used for estimation of volume of shale

- **Lithostratigraphic correlation**

Lithostratigraphic correlation was done by matching identical well log characteristic(s) across the five (5) different wells for the identification of the 25 payzones. The Gamma log (GR) as a lithology log was

mainly used for this purpose. Sand zones showed low gamma ray while shale zones showed high gamma rays. Radioactive sands however showed variant incursions of gamma rays or shales as expected depending on the severity of the zones. The GR log was placed side by side with the resistivity logs for all the wells for effective correlations to identify the sand zones and/or formation fluids characterization.

- **Reservoirs, fluid and fluid contact identification.**

The reservoirs were delineated by lithostratigraphic correlation across the five wells using the gamma log with the resistivity logs and neutron/density crossover as a guide. The reservoirs, fluids and fluid contact were done by correlating areas with good sand responses on the gamma log, high resistivity on the resistivity log and a crossover response from the neutron-density crossover.

- **Quantitative computation of well logs**

Quantitative computations of linear and non linear correlations of volumes of shales from gamma ray logs, net to gross ratios, effective porosity, formation volume factors, water saturation from the Archie's equation and permeability were done by using the petrophysical equations below.

- (i) **Linear correlation of the volume of shale ( $V_{sh}$ )**

The gamma ray log used to calculate volume of shale in porous reservoirs are expressed as decimal fraction or percentage and represented as  $V_{sh}$ . The calculation of gamma ray index is the first step needed to determine the volume of shale from gamma ray log. The linear response ( $V_{sh} = I_{GR}$ ) is shown in equation 1 below (Asquith and Krygowski, 2004).

$$\text{Volume of shale } (V_{sh}) = I_{GR} = \frac{GR_{log} - GR_{min}}{GR_{max} - GR_{min}} \quad (1)$$

Where, GR = Gamma log,  $GR_{min}$  = Minimum gamma log reading for sand and  $GR_{max}$  = Maximum gamma log reading for shale

- (ii) **Non Linear correlations of volume of shale ( $V_{sh}$ )**

Volume of shale using the Larinov (1969) correlation for tertiary shaly sand rocks is given below ;

$$V_{sh} = 0.083(2^{3.71I_{GR}} - 1) \quad (2)$$

Volume of shale using the Larinov (1969) correlation for older shaly sand rocks is given below;

$$V_{sh} = 0.33(2^{2I_{GR}} - 1) \quad (3)$$

Volume of shale using the Steiber (1970) correlation is shown in equation 4 below;

$$V_{sh} = \frac{I_{GR}}{3 - 2I_{GR}} \quad (4)$$

Volume of shale using the Clavier (1971) correlation is shown in equation 5 below;

$$V_{sh} = 1.7 - [3.38 - (I_{GR} + 0.7)^2]^{\frac{1}{2}} \quad (5)$$

- (iii) **Porosity ( $\Phi$ )**

This is the ratio of the total volume of void rock body to its bulk volume expressed as a percentage. Porosity may be evaluated in various ways but in this research it was determined from the density log with the aid of the software calculator using this equation;

$$\text{porosity } (\Phi) = \frac{2.65 - \rho_{log}}{2.65 - 1} \quad (6)$$

Where,  $\rho_{log}$  = density from density log.

**(iv) Total porosity ( $\phi_T$ )**

The total porosity of a porous rock is defined as the ratio of the entire pore space in a rock to its bulk volume. It was determined in this study from the neutron density log with the aid of the software calculator using this equation;

$$\text{Total porosity } (\phi_T) = \frac{\phi_D - NPFI}{2} \quad (7)$$

(Worthington, 1998). Where  $(\phi_T)$  = porosity and  $NPFI$  = Neutron porosity log

**(v) Effective porosity ( $\phi_E$ )**

The effective porosity of a porous rock is defined as the ratio of the part of the pore volume where the water can circulate to the total volume of a representative sample of the rock. It was computed by the software calculator using the equation;

$$\text{Effective porosity } (\phi_E) = \phi_T - (V_{sh} \times \phi_T) \quad (8)$$

(Worthington, 1998). Where  $\phi_E$  = Total porosity and  $V_{sh}$  = Volume of shale

**(vi) Formation factor**

The porosities for each sand zones identified were used to estimate the formation factors using the Archie's model as shown in equation 9 below.

$$F = a\phi^{-m} = \frac{R_o}{R_w} \quad (9)$$

Where;  $a$  = tortuosity factor ( $a = 1$ );  $R_o$  is the resistivity of the rock filled with only water ( $S_w = 1$ ).  $m$  = cementation factor and  $m = 1.3$  for unconsolidated sands and  $m = 1.8$  to  $2.5$  for consolidated sands.

**(vii) Water saturation ( $S_w$ )**

Water saturation analysis was calculated using water saturation from the Archie's model (Archie, 1942) as shown in equation 10.

$$S_w = \sqrt{\frac{F \times R_w}{R_t}} \quad (10)$$

Where;  $R_w$  = resistivity of the formation water;  $R_t$  = true formation resistivity;  $F$  = formation factor. The resistivity value in the water zone will be obtained from the resistivity log at the point of oil-water contact while the resistivity in oil zone was obtained from the resistivity log.

## Results

### Petrophysical results of linear, non-linear and modified volume of shale

Results of linear, non-linear and modified volume of shale against gamma ray shows that all the volumes reduced/declined inversely with increasing gamma ray content but the linear correlation (IGR) as expected gave an inverse straight line while all the non-linear correlations gave slightly inverted curves as shown in Fig. 3. It was also observed that there was a convergence at  $V_{sh} = 1.0$  at 10 gapi, while a noticeable divergence occurred with increased gamma ray and reduced volume of shale. This perhaps indicates that both correlations may not produce lower regressions of accuracies.

Consequently, this perceived assertion gave rise to the estimation of a modified volume of shale with gamma ray from the 25 payzones. Results gave similar inverse curve patterns with a correlation of  $V_{shmod} = -0.46\ln(GR_{log}) + 1.997$  with  $R^2 = 0.996$  as shown in Fig. 4.

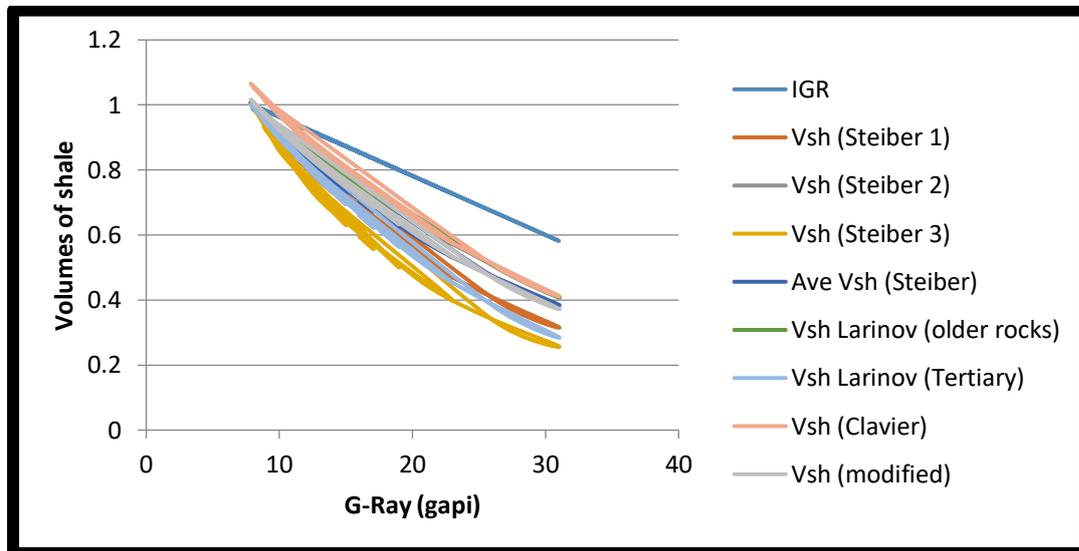


Figure 3: Result of linear, non-linear and modified  $V_{sh}$  against  $GR_{log}$

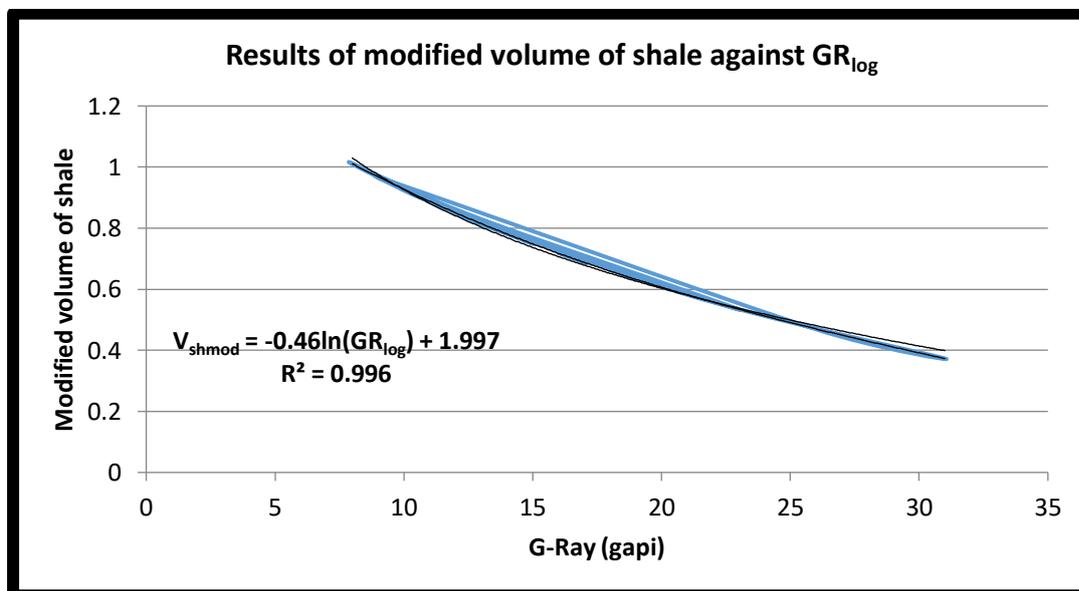
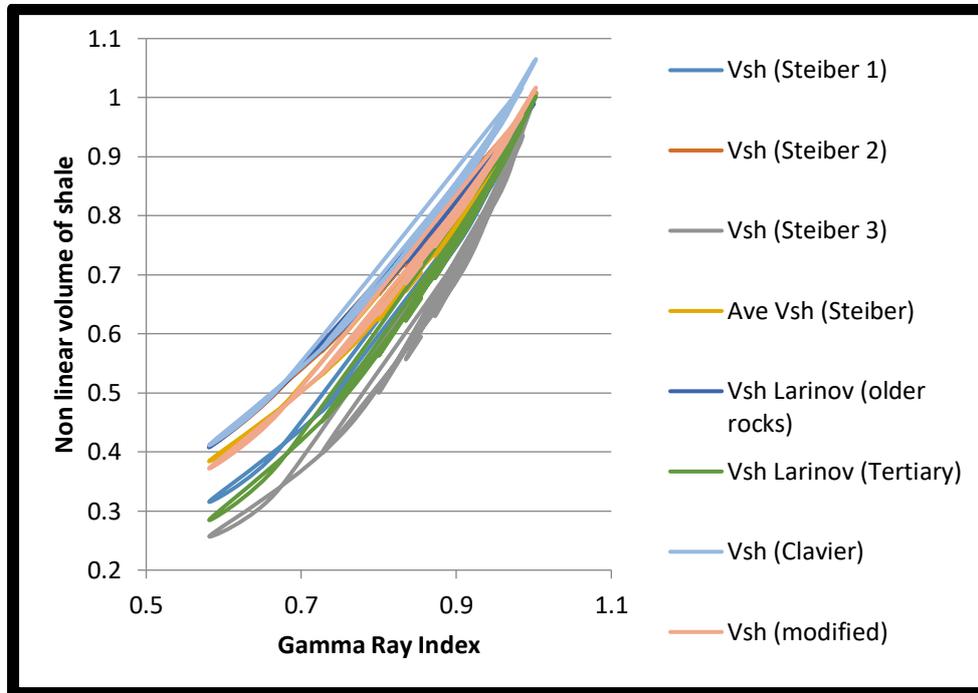


Figure 4: Results of modified volume of shale against  $GR_{log}$

This indicates that an approximation of both linear and non-linear correlations perhaps would give better resolution accuracy with appreciable regression.

Results of non-linear volumes of shale with linear volumes of shale all gave slightly increasing curves and converged at almost an IGR value of 1.0 as shown in Fig. 5. A modification of this result lies midway in the clusters. This indicates that an approximate model perhaps would also be appropriate.



**Figure 5:** Result of non-linear  $V_{sh}$  against gamma ray index (IGR) (linear)

### Results of modified volume of shale with non-linear and linear models

Results of modified volume of shale ( $V_{sh}$ ) against  $V_{sh}$  (Steiber 2) gave an increasing straight line almost from the origin with a correlation of  $V_{shmod} = 1.092V_{sh(Steiber)} - 0.089$  with  $R^2 = 0.999$  as shown in Fig. 6. Results of modified volume of shale ( $V_{sh}$ ) against  $V_{sh}$  (Steiber 3) gave increasing slightly curved straight line from the origin with a correlation of  $V_{shmod} = 0.828V_{sh(Steiber 3)} + 0.206$  with  $R^2 = 0.991$  as shown in Fig. 7. Results of modified volume of shale against  $V_{sh}$  (Average Steiber) gave an appreciable increasing straight line from the origin with a correlation of  $V_{shmod} = 1.034V_{sh(Average Steiber)} - 0.014$  with  $R^2 = 0.999$  as shown in Fig. 8.

Results of modified volume of shale against  $V_{sh}$  (Larinov-Older rocks) gave also an appreciable increasing straight line from the origin with a correlation of  $V_{shmod} = 1.114V_{shLO} - 0.105$  with  $R^2 = 0.997$  as shown in Fig. 9. Results of modified volume of shale against  $V_{sh}$  (Larinov-Tertiary rocks) gave an increasing straight line from the origin with a correlation of  $V_{shmod} = 0.885V_{shLT} + 0.130$  with  $R^2 = 0.999$  as shown in Fig. 10. Results of modified volume of shale against  $V_{sh}$  Clavier gave an increasing straight line from the origin with a correlation of  $V_{shmod} = 0.99V_{shClavier} - 0.035$  with  $R^2 = 0.999$  as shown in Fig. 11. Results of modified volume of shale against the linear  $V_{sh}$  (IGR) gave an increasing exponential curve from the origin with a correlation of  $V_{shmod} = 0.095e^{2.355(IGR)}$  with  $R^2 = 0.999$  as shown in Fig. 12.

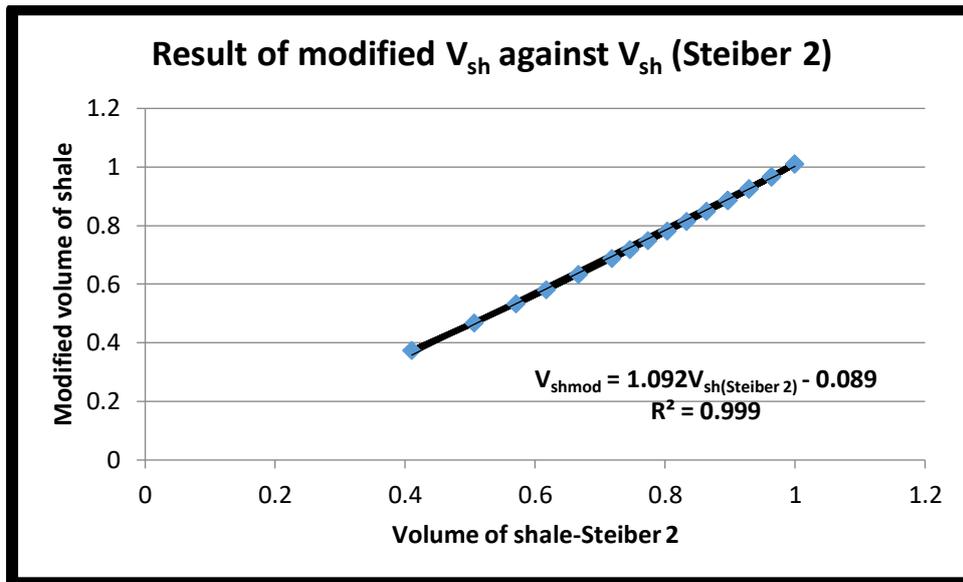


Figure 6: Result of modified  $V_{sh}$  against  $V_{sh}$  (Steiber 2)

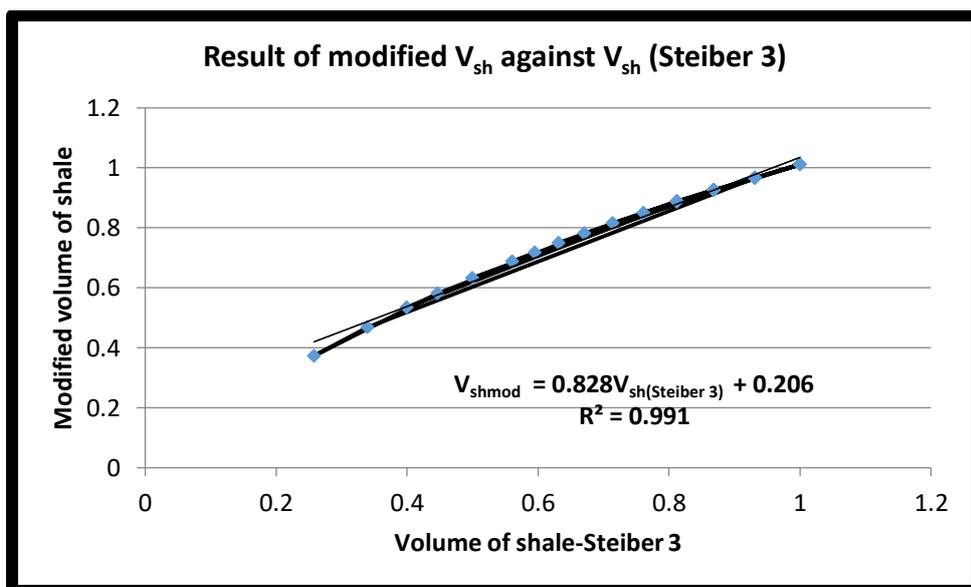


Figure 7: Result of modified  $V_{sh}$  against  $V_{sh}$  (Steiber 3)

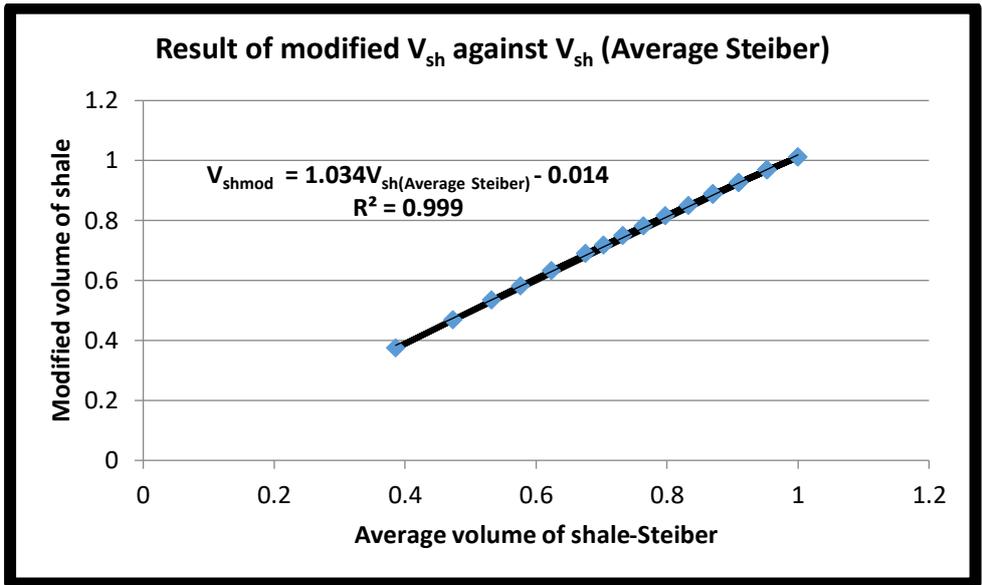


Figure 8: Result of modified  $V_{sh}$  against  $V_{sh}$  (Average Steiber)

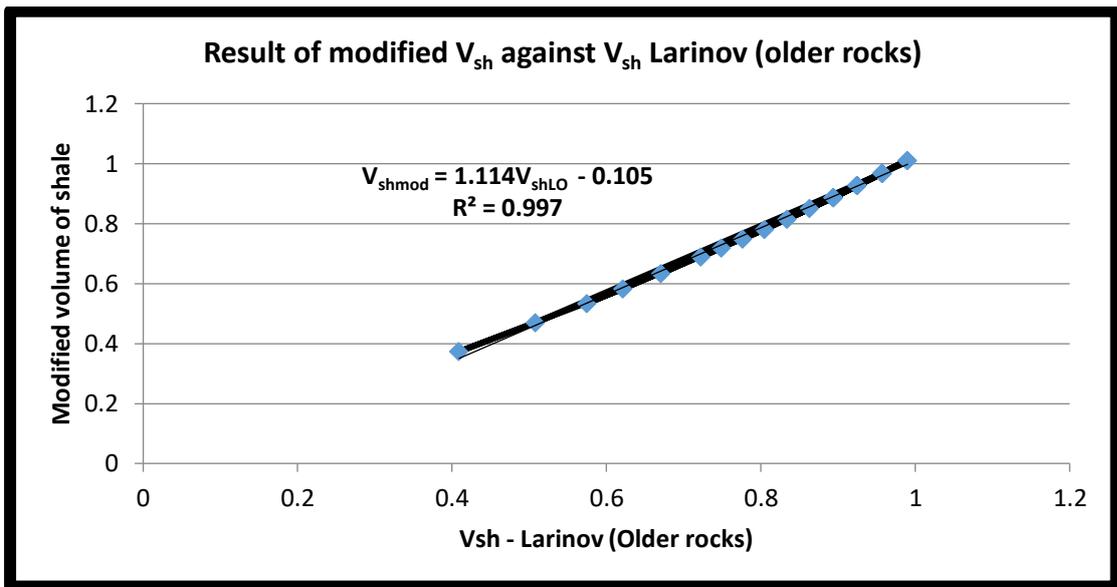


Figure 9: Result of modified  $V_{sh}$  against  $V_{sh}$  Larinov (older rocks)

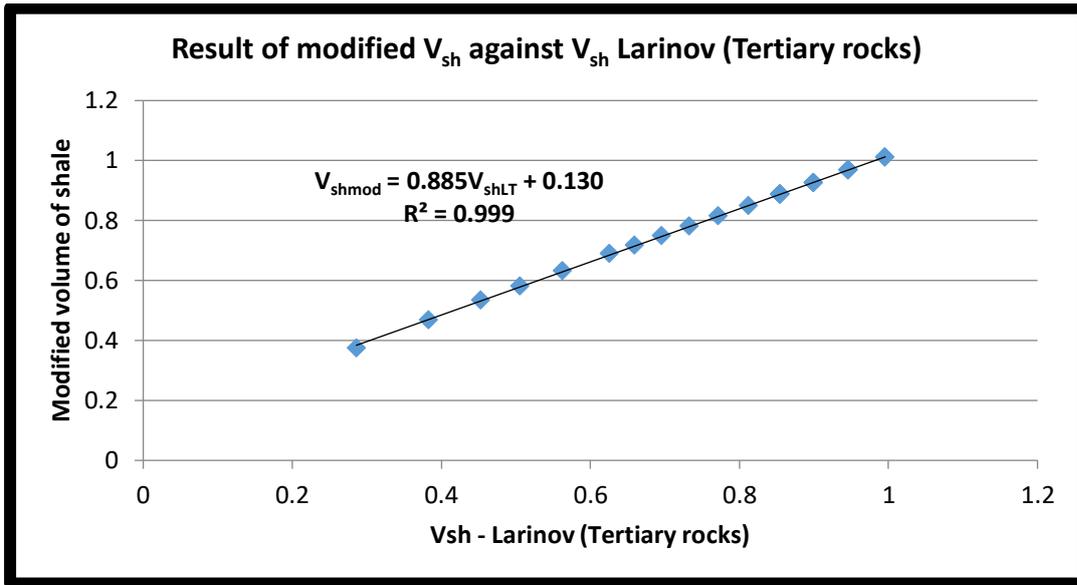


Figure 10: Result of modified  $V_{sh}$  against  $V_{sh}$  Larinov (Tertiary rocks)

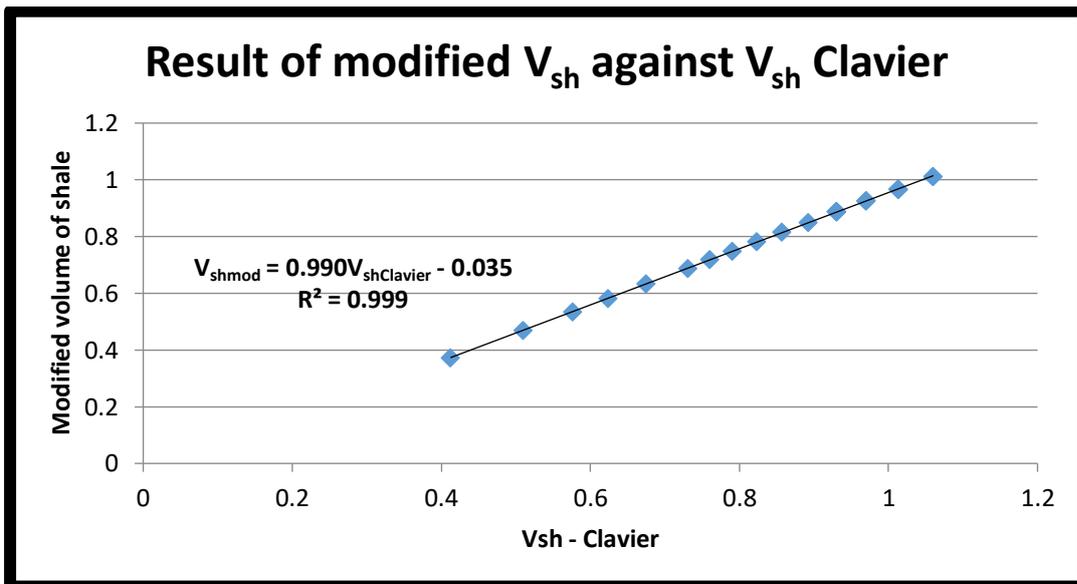
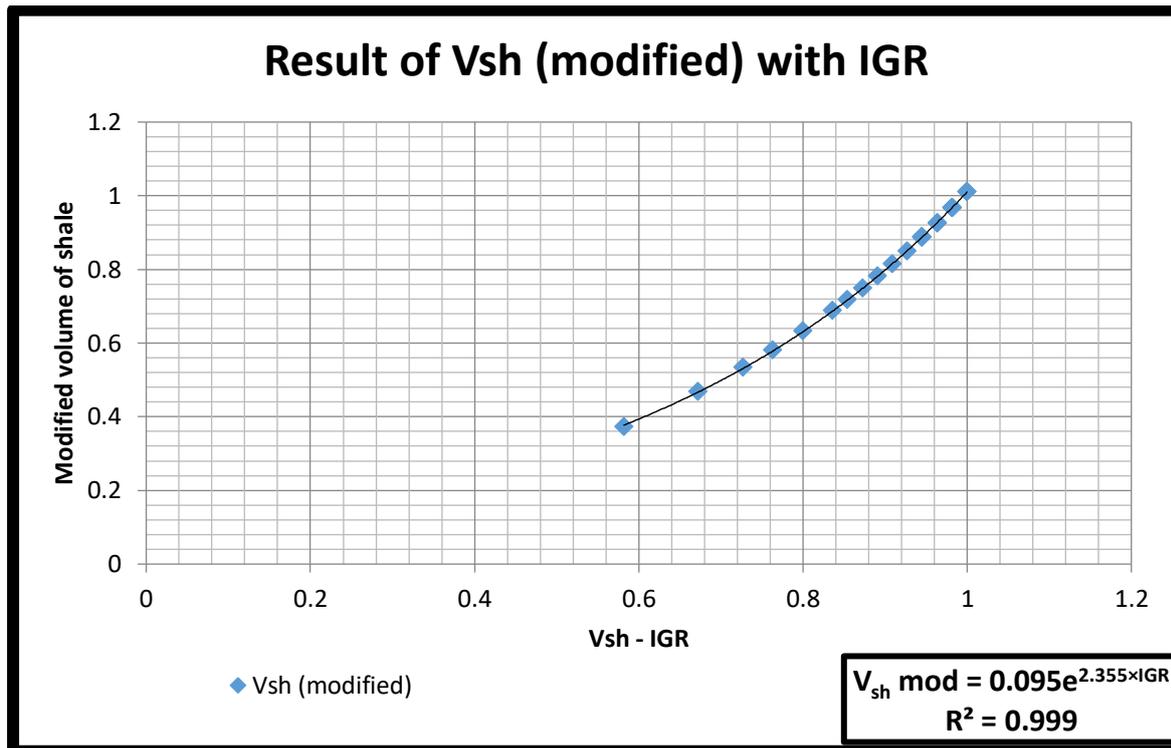


Figure 11: Result of modified  $V_{sh}$  against  $V_{sh}$  Clavier



**Figure 12:** Result of modified  $V_{sh}$  against  $V_{sh}$  IGR

### Summary

A total of seven new correlations were obtained from the comparative modification of results with linearly increasing trends when graphical comparisons were made between the non linear and modified volume of shale. The square regression estimations are appreciable with a minimum of 0.9 for all correlations.

However, six of these results gave approximately straight line patterns while one result gave a curve which depicts a non linear trend. This perhaps may indicate the severity of clay or shale content in the formation.

Critical reassessment of modified volume of shale against volume of shale (Larinov and older rocks) showed that its predicted model had the highest gradient (1.114) while the maximum intercept for all models was  $V_{sh}$  (Steiber 3). These variations are probably due to variations of reservoir characteristics where the studies were done. The modified  $V_{sh}$  performed well with appreciable square of regressions and can be applied for all estimation of volume of shales in this field.

### Conclusion

Detailed analysis of results shows that the linear model is a straight line while all the non-linear models are curves. However, modification of the non-linear models gave curves and appreciable regressions. A combination of linear and non-linear model approximation on the other hand also gave good regression. The linear model from this study only tends to slightly stretch out the non-linear curves. The resultant modified volume of shale correlation gave good match with the non-linear models such as the Steiber 2, average Steiber, Larinov (older rocks), Larinov (tertiary rocks) and Clavier.

Interestingly, this new composite model gave an exponential curve with the linear model (IGR), but with an appreciable regression. This perhaps indicates that the final model can perhaps be adopted in the field of study for wells and future wells to be drilled.

It is much clearer that the linear correlation modification comparison with non linear correlation gave effective interpretation and stands as a technical yard stick to quickly ascertain the severity of clay/shale contents. This correlation re-evaluation should be done during well log interpretation as an initial indicator of formation porosity. As a result of this, any deviation from linearity should be regarded as low porosity, low permeability and high shale content.

### Recommendation

1. It is recommended that more research be done in future developments of non linear correlations particularly for indigent wells and Fields in the Niger Delta.

### CONFLICTS OF INTEREST

There are no conflicts to declare.

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